Chapter 8
Similarity

Section 5
Proving Triangles are Similar

GOAL 1: Using Similarity Theorems

In this lesson, you will study two additional ways to prove that two triangles are similar: the Side-Side-Side (SSS) Similarity Theorem and the Side-Angle-Side (SAS) Similarity Theorem. The first theorem is proved in Example 1 and you are asked to prove the second theorem in Exercise 31.

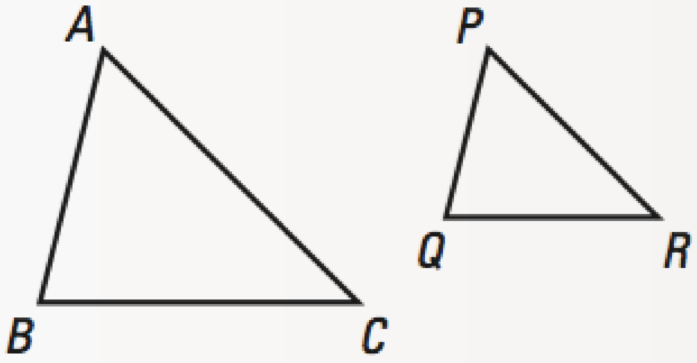
THEOREMS

THEOREM 8.2 ***Side-Side-Side (SSS) Similarity Theorem***

If the lengths of the corresponding sides of two triangles are proportional, then the triangles are similar.

If $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP}$,

then $\triangle ABC \sim \triangle PQR$.

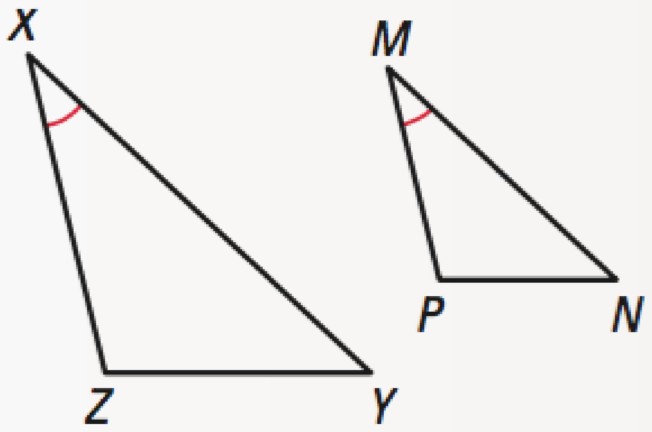


THEOREM 8.3 ***Side-Angle-Side (SAS) Similarity Theorem***

If an angle of one triangle is congruent to an angle of a second triangle and the lengths of the sides including these angles are proportional, then the triangles are similar.

If $\angle X \cong \angle M$ and $\frac{ZX}{PM} = \frac{XY}{MN}$,

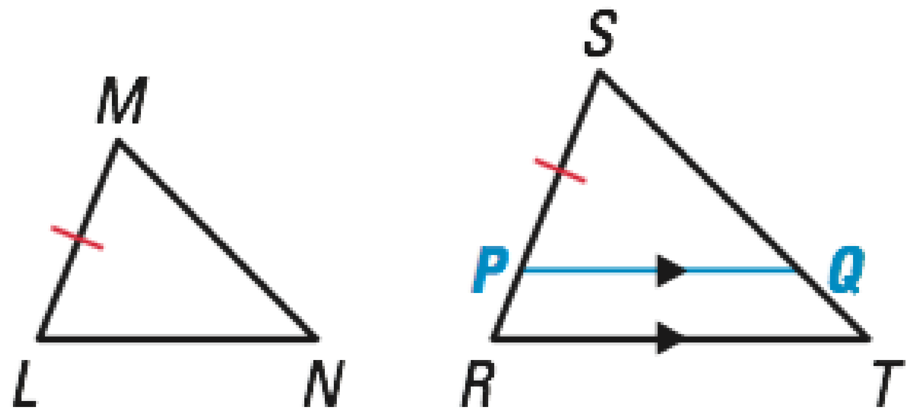
then $\triangle XYZ \sim \triangle MNP$.



Example 1: Proof of Theorem 8.2

GIVEN ▶ $\dfrac{RS}{LM} = \dfrac{ST}{MN} = \dfrac{TR}{NL}$

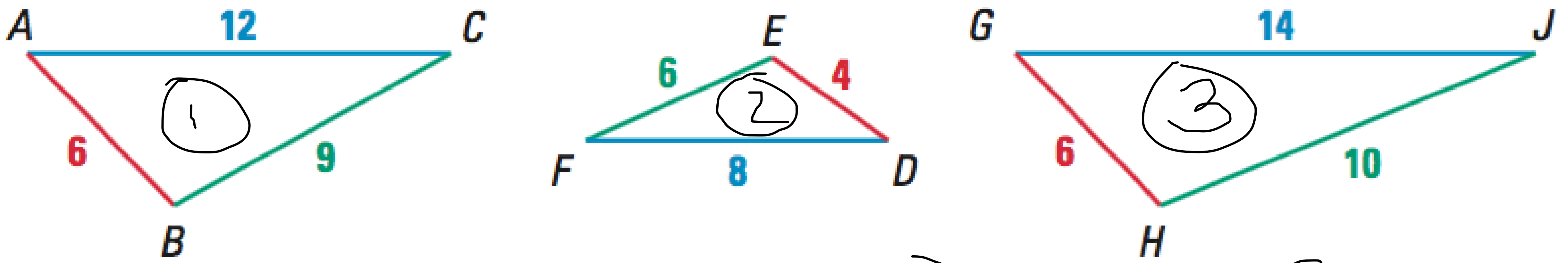
PROVE ▶ $\triangle RST \sim \triangle LMN$



Paragraph Proof Locate P on $\overline{RS}$ so that $PS = LM$. Draw $\overline{PQ}$ so that $\overline{PQ} \parallel \overline{RT}$. Then $\triangle RST \sim \triangle PSQ$, by the AA Similarity Postulate, and $\dfrac{RS}{PS} = \dfrac{ST}{SQ} = \dfrac{TR}{QP}$. Because $PS = LM$, you can substitute in the given proportion and find that $SQ = MN$ and $QP = NL$. By the SSS Congruence Theorem, it follows that $\triangle PSQ \cong \triangle LMN$. Finally, use the definition of congruent triangles and the AA Similarity Postulate to conclude that $\triangle RST \sim \triangle LMN$.

Example 2: Using the SSS Similarity Theorem

Which of the following three triangles are similar?



1 & 2

$6/4 \rightarrow 3/2$

$12/8 \rightarrow 3/2$

$9/6 \rightarrow 3/2$

yes

2 & 3

$4/6 \rightarrow 2/3$

$8/14 \rightarrow 4/7$

no

1 & 3

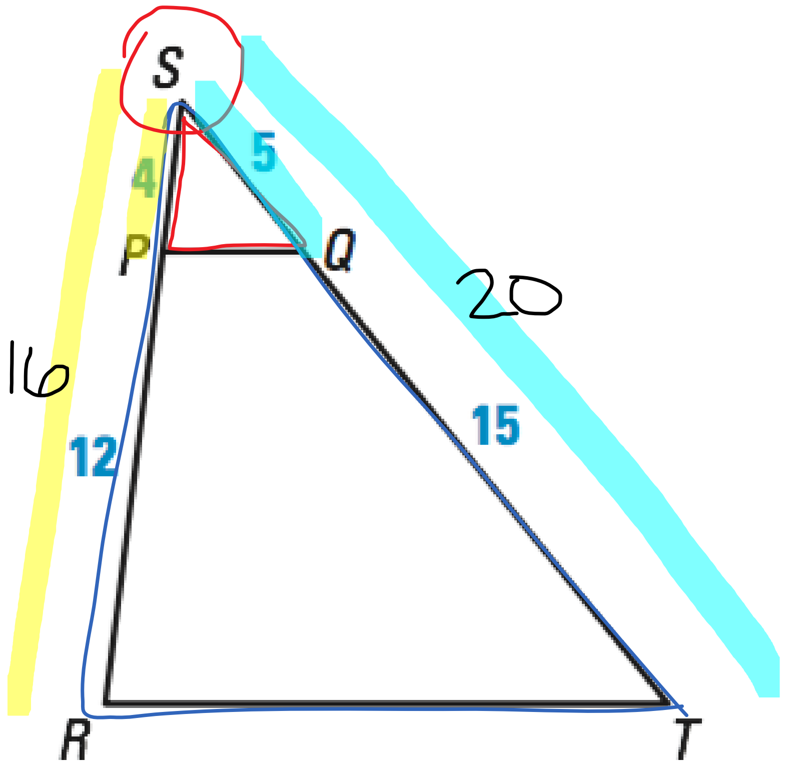
$6/6 \rightarrow 1/1$

$12/14 \rightarrow 6/7$

no

Example 3: Using the SAS Similarity Theorem

Use the given lengths to prove that $\Delta RST \sim \Delta PSQ$.



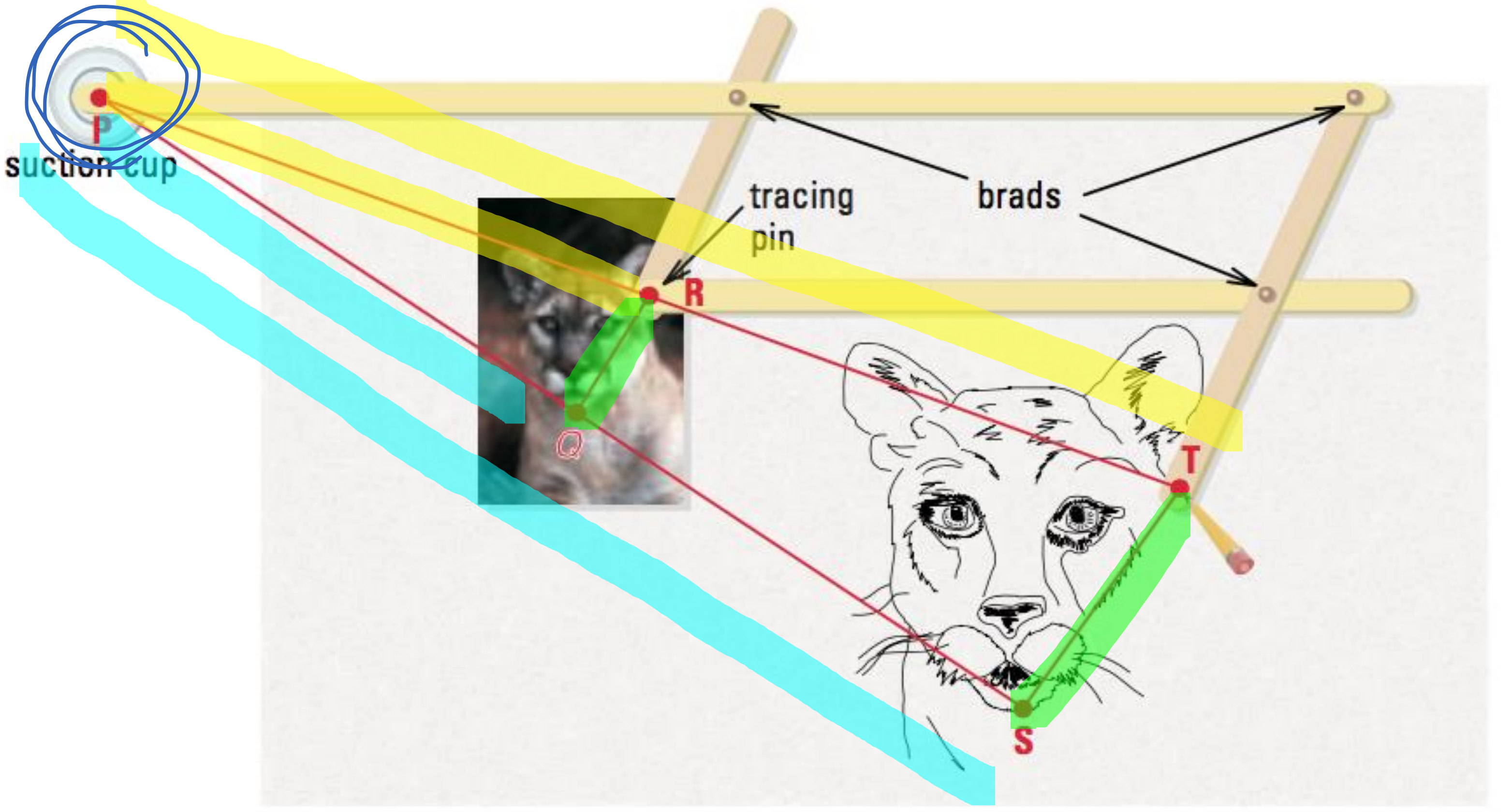
$\frac{4}{16} \rightarrow \frac{1}{4}$ $\angle S \cong \angle S$ $\frac{5}{20} \rightarrow \frac{1}{4}$

$\Rightarrow$ similar by SAS

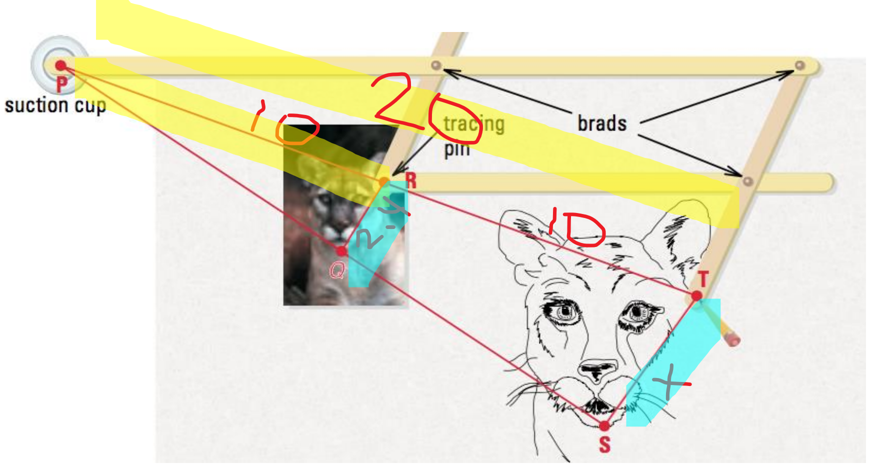
GOAL 2: Using Similar Triangles in Real Life

Example 4: Using a Pantograph

SCALE DRAWING As you move the tracing pin of a *pantograph* along a figure, the pencil attached to the far end draws an enlargement. As the pantograph expands and contracts, the three brads and the tracing pin always form the vertices of a parallelogram. The ratio of PR to PT is always equal to the ratio of PQ to PS. Also, the suction cup, the tracing pin, and the pencil remain collinear.



Example 4: Using a Pantograph (continued)



a) How can you show that $\Delta PRQ \sim \Delta PTS$?

Can use SAS → PR/PT = PQ/PS and <P cong. <P

b) In the diagram, PR is 10 inches and RT is 10 inches. The length of the cat, RQ, in the original print is 2.4 inches. Find the length of TS in the enlargement.

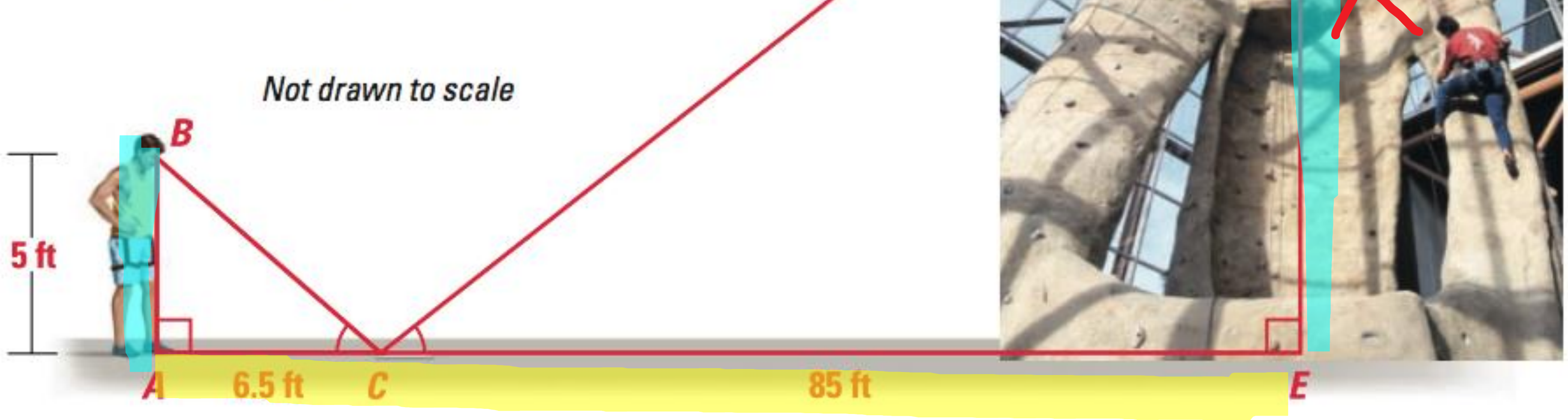
$$\frac{10}{20} = \frac{2.4}{x}$$

$$\frac{\cancel{10}x}{\cancel{10}} = \frac{48}{10} \rightarrow x = 4.8 \text{ in.}$$

Similar triangles can be used to find distances that are difficult to measure directly. One technique is called *Thales' shadow method* (page 486), named after the Greek geometer Thales who used it to calculate the height of the Great Pyramid.

Example 5: Finding Distance Indirectly

ROCK CLIMBING You are at an indoor climbing wall. To estimate the height of the wall, you place a mirror on the floor 85 feet from the base of the wall. Then you walk backward until you can see the top of the wall centered in the mirror. You are 6.5 feet from the mirror and your eyes are 5 feet above the ground. Use similar triangles to estimate the height of the wall.



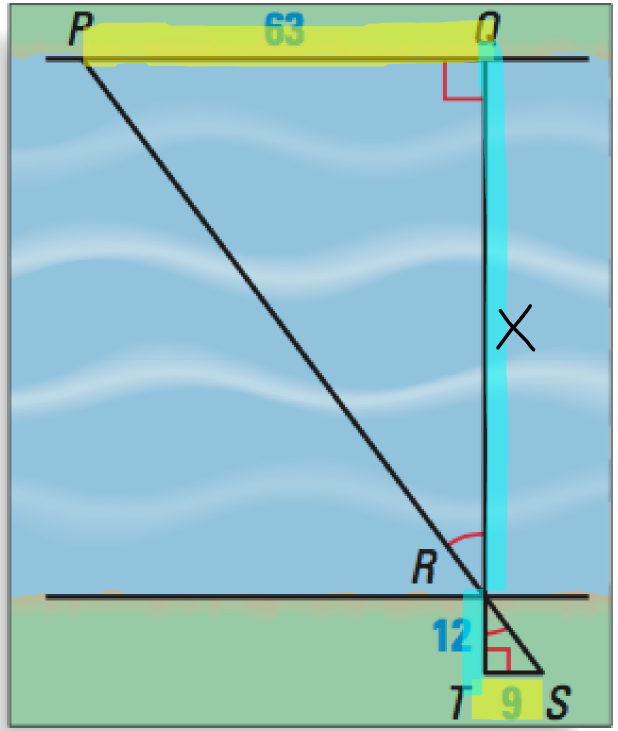
$$\frac{6.5}{85} = \frac{5}{x}$$

$$\frac{\cancel{6.5}x}{\cancel{6.5}} = \frac{425}{6.5}$$

$$x = 65.4 \text{ ft}$$

Example 6: Finding Distance Indirectly

INDIRECT MEASUREMENT To measure the width of a river, you use a surveying technique, as shown in the diagram. Use the given lengths (measured in feet) to find *RQ*.



$$\frac{9}{63} = \frac{12}{X}$$

$$\frac{\cancel{9}X}{\cancel{9}} = \frac{756}{9}$$

$$X = 84 \text{ ft}$$

EXIT SLIP